Mechanics of Generalized Continua with Application to Multiscale Phenomena

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The classical Cauchy continuum mechanics can be extended to incorporate additional microstructural features of materials and account for some observed size effects in the materials' behaviour. Two main enhancements are possible and will be presented in two lectures:

- **Gradient Theories.** The size–independent continuum mechanics of materials is based on the strain tensor, i.e. the symmetric part of the gradient of the displacement field. Introducing the second and third gradients of the displacement field makes it possible to describe size–dependent stress concentration, and surface tension effects. The corresponding theories elaborated by Mindlin [1] will be presented and applied to molecular dynamics effects in crystalline solids. Some size effects observed in the plastic behaviour of metals can be modelled by introducing the dislocation density tensor which is directly related to the gradient of plastic strain. This extension of crystal plasticity will be presented based of the framework established by Gurtin [2].

- **Micromorphic Media.** The classical continuum can also be extended by introducing additional degrees of freedom representing the deformation of microstructural elements. The micromorphic model by Eringen and Mindlin represents a versatile theory that can be applied to the size–dependent mechanics of composites and of plastic crystalline solids. In composites, the micromorphic theory is applicable at the limit of the hypothesis of separation of scale. In crystalline solids, the micromorphic model is closely related to the previous strain gradient plasticity models [3].

References

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[2] Gurtin M.E. (2003). *On a framework for small–deformation viscoplasticity: free energy, microforces, strain gradients.* International Journal of Plasticity, vol. 19, pp 47–90.

[3] Forest S. (2008). *Some links between Cosserat, strain gradient crystal plasticity and the statistical theory of dislocations.* Philosophical Magazine, vol. 88, pp 3549–3563.