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Chapter 3: Homogenization methods for media with simple and complex behaviours (elastic, elastic-plastic, visco-plastic, elastic-viscoplastic heterogeneous materials)

The chapter 3 is dedicated to homogenization techniques based on the calculation of the Eshelby inclusion (seen Chapter 2). In the first section, we develop different methods of determining the effective properties of elastic heterogeneous materials. We analyse more precisely the Voigt model, the Reuss model, the Eshelby model, the Mori-Tanaka model and the classical self-consistent model. We compare the results in a two phase elastic composite with incompressible phases to assess the validity and accuracy of these methods.

If one or more of the heterogeneities in a heterogeneous medium is no more linear elastic (elastoplastic, elasto-viscoplastic, etc.), the overall behaviour will be non linear. In these cases, the determination of the overall properties is more complex. The main difficulty is to well-model the elastic-inelastic mechanical interactions between the heterogeneities. Historically, the homogenization methods for inelastic behaviour were developed mainly for crystalline materials. The grains (single-crystal) were considered as the heterogeneities and the aim was to relate the mechanical response of a polycrystal to the fundamental mechanisms of single-crystal deformation. The second section of the chapter deals with the homogenization techniques for inelastic behaviours. First, we establish the constitutive equations for single-crystal using crystal plasticity theory and a viscoplastic framework. Second, we introduce various homogenization methods to relate the single-crystal behaviour to the polycrystal's one accounting for the mechanical interactions between grains. These homogenization methods are extended from the linear elastic case of the inelastic cases using linearization procedures (tangent, affine). Third, we finally discuss the case of linear and non linear elastoviscoplastic behaviours where complex time-space couplings involved so that long memory effect appears.