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Chapter 1: Fundamental equations of the micromechanics

This first chapter introduces the different possible microstructures of heterogeneous materials. The different origins of thermo-mechanical inhomogeneities are highlighted like the chemical, physical, and, crystallographic inhomogeneities, or, lattice defects. Then, different classes of heterogeneous materials are presented (composites, polycrystalline, multiphase materials), as well as, the topological and morphological description, the notion of representative volume element (RVE), and, the volume fractions. In this lecture, the basic equations of continuum mechanics used in micromechanics of solids are also recalled such as: stresses, equilibrium conditions, boundary conditions, volume forces. The kinematics in the small perturbation hypothesis is recalled (Displacement, distortions, rotations, compatibility conditions). Fundamental energy principles (Minimum energy principle, complementary energy principle) and Mandel-Hill condition are introduced.

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Chapter 2: Heterogeneous linear elastic media with defects (eigenstrains)

In the second chapter, the relevant notion of "eigenstrain" is introduced and a complete formulation for an infinite heterogeneous linear elastic solid with eigenstrains is presented. The formulation is written under Navier type partial differential equations, and, the transformation into an integral equation using Green functions for infinite homogeneous media is described. The essential properties of Green functions and Kröner modified Green functions are detailed.

This general formulation allows treating two elementary problem applications:
The first one is the problem of heterogeneous and incompatible ellipsoidal inclusions (Eshelby, Kröner,) leading to the estimations of 2nd order internal stresses in a heterogeneous material; The second one is the derivation of the distortion field of dislocations like pure edge, screw dislocations or dislocation loops.

Some applications useful in materials and composite sciences are also described through pile-ups of dislocation loops at grain boundaries, pairs of inelastic inclusions, interfacial operators and coated inclusions.